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NEW APPROACH ON BINARY SUPRA MULTISET TOPOLOGICAL SPACE

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Abstract: In this paper, we introduce the concept of binary supra M-topology (bsm), which combines features of binary supra topology and supra M-topology. Several concrete examples are presented to illustrate the structure and behavior of binary supra M-topology. Additionally, various properties of binary supra M-topology are explored and analyzed.

Keywords and Phrases: Binary supra M-topological space, binary supra open multiset, binary supra closed multiset, binary supra interior multiset, binary supra closure multiset.

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1. Introduction

A multiset is regarded as a generalization of a set in mathematics. According to classical set theory, a set is a clearly defined collection of unique items. If an object can appear more than once in a set, then the mathematical structure is called a *multiset* [2]. In a topological context, a multiset considers the number of occurrences of an element x in a multiset M. We represent the multiset M drawn from the set $X = \{x_1, x_2, \ldots, x_n\}$ as $M = \{m_1/x_1, m_2/x_2, \ldots, m_n/x_n\}$, where m_i is the number of occurrences of the element x_i (for $i = 1, 2, \ldots, n$) in the multiset M.The concept of an M-topological space has been investigated through multiset closure, multiset interior, multiset limit points, multiset neighborhoods, and multiset continuous functions [7]. A set X with a family μ of its subsets is called a supra topological space, denoted by (X, μ), if $X \in \mu$ and the arbitrary union of members of μ are examined. Devi, et. al, [3] established the binary properties of open and closed are discussed. Elekiah established the concept of some properties of S_α open and closed sets in binary topological space [4] and also El-Sheikh [6] introduced the supertopological space properties of decompositions of some types supra supramultisets. Mashhour, et.al., [10] presented the supra topological space some basic theorems and examples are examined. Assad [1] contributed to the study of some operators on supra topological spaces, El-Shafei [5] presented applications of pre-open sets in supra topological spaces, exploring supra boundaries and supra limit points with respect to pre-open sets, and examining their behavior in spaces with the difference property. A binary topological space [9] is a structure that simultaneously considers the subsets of sets X and Y, studying information about ordered pairs (A, B) of subsets of X and Y. In a generalized binary topological space, certain conditions involving intersections of elements of these subsets are satisfied. Lellis Thivagar [8] presented by on binary structure of supra topological space discussed continuous functions and their properties for binary supra α set, binary supra β set , binary supra semi-open set, binary supra pre-open set are discussed. Shravan and Tripathy [12], [13] presented the multiset topological space with kuratowski closure operators and metrizability related theorem and examples are discussed. In this paper, we introduce the concept of the *binary supra M-topological space* $(b_{\mu M})$. Additionally, we define and examine several types of binary supra multisets, including the binary supra open multiset, binary supra interior multiset, binary supra closure multiset, binary supra M-neighborhood, binary supra pre-open multiset, and binary supra α-open multiset. The properties of these binary supra multiset are also thoroughly investigated.

2. Preliminaries

Definition 2.1. [10] *A subfamily τ^* of X is said to be a supra topology on X if,*
(i) X, $\phi \in \tau^*$.
(ii) If $A_i \in \tau^*$ for all $i \in J$, then $\cup A_i \in \tau^*$.
(X, τ^*) is called a supra topological space. The elements of τ^* are called supra open sets in (X, τ^*) and complement of a supra open set is called a supra closed set.

Definition 2.2. [10] *The supra closure of a set A is denoted by supra cl(A) and defined as supra as supra cl(A)= $\cap\{B : B$ is a supra closed and $A \subseteq B\}$.*
The supra interior of a set A is denoted by supra int(A), and defined as supra int(A)= $\cup\{B : B$ is a supra open and $A \supseteq B\}$.

Definition 2.3. [10] *Let (X, τ) be a topological space and τ^* be a supra topology on X. We call τ^* a supra topology associated with τ if $\tau \subseteq \tau^*$.*

Definition 2.4. [10] *Let (X, τ^*) be a supra topological space. A set A is called supra semiopen sett if $A \subseteq supracl(supraint(A))$.*

Definition 2.5. [7] *Let $M \in [X]^w$ and $\tau \subseteq P^*(M)$. Then τ is called a multiset topological space of M if τ satisfies the following properties.*
(i) The multiset M and the empty multiset ϕ are in τ.
(ii) The multiset union of the elements of any sub collection of τ is τ.
(iii) The multiset Intersection of the elements of any finite sub collection of τ is inτ.

Definition 2.6. [7] *A sub multiset N of M-topological space M in $[X]^w$ is said to be closed if the multiset$M \ominus N$ is open. In discrete M-topological space every multiset is an open multiset as well as a closed multiset. In the M-topological space PF(M)$\cup\phi$, every multiset is an open multiset as well as a closed multiset.*

Definition 2.7. [7] *Given a subsmet A of an M-topological space M in $[X]^w$, the interior of A is defined as the multiset union of all open multiset contained in A and its denoted by $Int(A)$. i.e., Int(A)=$\cup\{G \subseteq M : G$is an open multiset and $G \subseteq A\}$ and $C_{Int(A)}(x) = max\{C_G(x) : G \subseteq A\}$.*

Definition 2.8. [7] *Given a subsmet A of an M-topological space space M in $[X]^w$, the closure of A is defined as the multiset intersection of all closed multiset containing A and its denoted by $Cl(A)$. i.e., $Cl(A) = \cap\{K \subseteq M : K$is a closed multiset and $A \subseteq K\}$ and $C_{Cl(A)}(x) = min\{C_K(x) : A \subseteq K\}$.*

Definition 2.9. [7] *Let (M, τ) be a M-topological space, let $x \in E^k M$ and $N \subseteq M$. Then N is said to be a neighborhood of k/x if there is an open multiset V in τ such that $x \in^k V$ and $C_V(y) \leq C_N(y)$ for all $y \neq x$. i.e., a neighborhood of k/x in M means any open multiset containing k/x. Here k/x is said to be an interior points of N.*

Definition 2.10. [9] *Let X And Y be any two no empty sets. A binary topological space X to Y is a binary structure $M \subseteq \rho(X) \times \rho(Y)$ that satifies the folowing axioms.*
(i) (ϕ, ϕ) and(M,N) $\in$ M.
(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in M$ whenever $(A_1, B_1) \in M, (A_2, B_2) \in M$.
(iii) If $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\}$ is a family of members of M, then
$(\bigcup_\alpha \in \Delta A_\alpha, \bigcup_\alpha \in \Delta B_\alpha) \in$M.

Definition 2.11. [8] *Let μ be a collection of subsmets of X. Then, $\mu \subseteq P^*(X)$ is called supra M-topology on X if the following conditions are satisfied:*
(i) X, $\Phi \in \mu$.
(ii) The union of any number of multisets in μ belongs to μ.
The pair (X, μ) is called supra M-topological space (or supra M-spaces) over X.

Definition 2.12. [8] *A subset A of X is called*
(i) supra semiopen if $A \subseteq cl_\mu(int_\mu(A))$.
(ii) supra preopen if $A \subseteq int_\mu(cl_\mu)$.
(iii) supra α-open if $A \subseteq int_\mu(cl_\mu(int_\mu(A)))$.

3. Binary Supra M-topology

In this section, the binary supra multiset (bsm) and some preoperties are discussed.

Definition 3.1. *A binary supra multiset topological space (briefly, binary supra M-topological space) from M to N is represented as, $b_{\mu M} \subseteq P^*(M) \times P^*(N)$, where $b_{\mu M}$ satisfies the following axioms.*
(i) $(M, N) \in b_{\mu M}$ and$(\phi, \phi) \in b_{\mu M}$.
(ii) For any family $\{(A_\alpha, B_\alpha) : \alpha \in \Delta\} \subseteq b_{\mu M}$, $(\cup A_\alpha, \cup B_\alpha) \in b_{\mu M}$.
If $b_{\mu M}$ forms a binary supra M-topology from M to N, then $(M, N, b_{\mu M})$ is termed a binary supra M-topological space. The elements of $b_{\mu M}$ are referred to as binary supra open multiset and their complements are binary supra closed multiset.

Definition 3.2. *Let $(M, N, b_{\mu M})$ be a binary supra M-topological space and $(M_1, N_1) \in M \times N$. A binary supra multiset (A, B) of (M,N) is a binary supra M- negihbourhood if there exists a binary supra open multiset (U,V) such that $(M_1, N_1) \in (U, V) \subseteq (A, B)$.*

Example 3.3. Let M={1/a, 2/b, 3/c} and N={1/a, 1/b} and $b_{\mu M} = \{(M, N), (\phi, \phi), (\{1/b\}, \{1/a\}), (\{2/c\}, \phi)\}$. Then $(M, N, b_{\mu M})$ forms binary supra M-topology.

Definition 3.4. *Let $(M, N, b_{\mu M})$ be a binary supra M-topology over M and N. The members of $b_{\mu M}$ are termed binary supra open multiset in M and N. The set of all binary supra open multiset over M and N is denoted by BSMO $(M, N, b_{\mu M})$,and the set of all binary supra closed multiset is denoted by BSMC $(M, N, b_{\mu M})$.*

Definition 3.5. *In a binary supra M-topological space $(M, N, b_{\mu M})$, a multiset (Z_1, Z_2) is termed a binary supra closed multiset if its relative complement $(Z_1, Z_2)^C$ is a binary supra open multiset.*

Definition 3.6. *The collection of (M,N) and (ϕ, ϕ) forms a binary supra multiset indiscrete topological space or binary supra multiset trivial topological space*

$(M, N, b_{\mu M})$.

Note 3.7. Let binary supra multiset indiscrete topological space denoted as $[I = (M, N), (\phi, \phi)]$ then (M, N, I).

Definition 3.8. *The collection of all bsm power set of (M,N) if $(M, N, b_{\mu M})$ is an binary supra multiset discrete topological space (or) trivial topological space.*

Note 3.9. Let $D = P(M) \times P(N)$, be a binary supra multiset discrete topological space. The support bsm power set of (M, N, D) bsm discrete topological space (or) bsm trivial topological space.

Definition 3.10. *Let $b^1_{\mu M}$ and $b^2_{\mu M}$ be two non-empty binary supra multiset of (M,N). If $b^1_{\mu M}$ is said to be bsm coarser, weaker, or smaller than and $b^2_{\mu M}$ if every binary supra open multiset $b^1_{\mu M}$ is also a $b^2_{\mu M}$. Conversely, $b^2_{\mu M}$ is bsm finer, bsm stronger or bsm greater binary supra multiset topological space than $b^1_{\mu M}$. If Both $b^1_{\mu M}$ and $b^2_{\mu M}$ not bsm comparable for $b^1_{\mu M}$ $b^2_{\mu M}$ other wise is bsm comparable.*

Example 3.11. Let $b^1_{\mu M} = \{(M, N), (\phi, \phi), (\{1/x\}, \{1/y\})\}$, $b^2_{\mu M} = \{(M, N), (\phi, \phi), (\{2/x\}, \{2/y\})(\phi, N)\}$. Since $b^1_{\mu M}$ is bsm coarser than $b^2_{\mu M}$, or $b^2_{\mu M}$ is bsm finer than $b^1_{\mu M}$. They are bsm comparable topologies space.

Definition 3.12. *A binary supra M-topological space $(M, N, b_{\mu M})$ is termed binary supra M-neighborhood of the multi-points $(\{m/x\}, \{n/y\}) \in (M, N)$ if there exist a binary supra open multiset (G, H) such that $(\{m/x\}, \{n/y\}) \in (G, H) \subseteq (Z_1, Z_2)$. The binary supra M-neighborhood system of a binary supra multi-points $\{m/x\}$ and $\{n/y\}$, denoted by binary supra $b_{\mu M}$ is the family of all its binary supra M-neighborhood.*

Definition 3.13. *Let $(M, N, b_{\mu M})$ be a binary supra M-topological sapce and $(G, H) \in (P^*(M), P^*(N))$. Then binary supra interior multiset of (G,H) denoted $int_{\mu M}(G, H)$, is the union of all binary supra open sub multiset of (G,H). Formally, $int_{\mu M}(G, H) = \cup\{(I, J) : (I, J)$ is a binary supra open multiset and $(I, J) \subseteq (G, H)\}$ and $C_{int_{\mu M}}(M, N) = max\{C_{(I,J)}(M, N) : (I, J) \subseteq (G, H), (I, J) \in b_{\mu M}\}$.*

Definition 3.14. *Let $(M, N, b_{\mu M})$ be a binary supra M-topological space and $(K, L) \in (P^*(M), P^*(N))$. Then binary supra closure multiset of (K,L), denoted $cl_{\mu M}$ (K, L) is the intersection of all binary supra closed multiset containing (K,L). Formally, $cl_{\mu M}(K, L) = \cap\{(I, J) : (I, J)$ is a binary supra closed multiset and $(K, L) \subseteq (I, J)\}$ and $C_{cl_{\mu M}}(M, N) = min\{C_{(K,L)}(M, N) : (I, J) \subseteq (K, L), (I, J) \in b_{\mu M}\}$.*

Example 3.15. Let $M_1 = \{1/p, 1/q\}, M_2 = \{1/p, 2/q, 1/r\}$. We consider the binary supra M-topological space $b_{\mu M} = \{(\phi, \phi), (M, N), (\{1/p\}, \{2/q\}), (\phi, M),$

$(\{1/r\}, \{1/p\})\}$ and $(b_{\mu M})^C = \{(\phi, \phi), (M, N), (\{1/q\}, \phi), (\{1/r\}, \phi),$
$(\{1/p, 2/q\}, \phi)\}$. Clearly, is an $b_{\mu M}$ binary supra M-topological space
$(A, B) = (\{1/p, 1/r\})$, $int_{\mu M}(A, B) = (\{1/p, 1/r\})$ and $cl_{\mu M} = (\{1/p, 1/r\})$.

Definition 3.16. *The ordered pair* $((M, N)^{1*}, (M, N)^{2*})$ *is the binary supra closure multiset of (M,N) denoted by* $cl_{\mu M}(M, N)$ *in the binary supra M-topological space* $(M, N, b_{\mu M})$. *Here,* $(M, N) \subseteq (M, N)$.
(i) $(M, N)^{1*} = \bigcap\{M_\alpha : (M_\alpha, N_\alpha)$ *is binary supra closed multiset and* $(M, N) \subseteq (M_\alpha, N_\alpha)\}$,
(ii) $(M, N)^{2*} = \bigcap\{N_\alpha : (M_\alpha, N_\alpha)$ *is binary supra closed multiset and* $(M, N) \subseteq (M_\alpha, N_\alpha)\}$.

Definition 3.17. *Let* $(M, N, b_{\mu M})$ *be a binary supra M-topological space with* $(M, N) \subseteq (M, N)$. *The ordered pair* $((M, N)^{1o}, (M, N)^{2o})$ *is the binary supra interior multiset of (M,N) denoted by* $int_{\mu M}(M, N)$,
(i) $(M, N)^{1o} = \bigcup\{M_\alpha : (M_\alpha, N_\alpha)$ *is binary supra open multiset and* $(M, N) \subseteq (M_\alpha, N_\alpha)\}$,
(ii) $(M, N)^{2o} = \bigcup\{N_\alpha : (M_\alpha, N_\alpha)$ *is binary supra open multiset and* $(M, N) \subseteq (M_\alpha, N_\alpha)\}$.

Example 3.18. Let $M = \{2/a, 1/b, 3/c\}$ and $N = \{1/a, 2/b\}$ with binary supra M-topology given by, $b_{\mu M} = \{(M, N), (\phi, \phi), (\{1/a, 1/c\}, N), (\{2/a, 1/b\}, \{2/b\}),$ (ϕ, N) and its complement $(b_{\mu M})^C = \{(M, N), (\phi, \phi), (\{1/a,\ 1/b, 3/c\}, \phi), (M, \phi)$.
If $(M, N) = (\{1/b\}, \phi)$, then
$cl_{\mu M}(M, N) = (\{1/a, 1/b, 3/c\}, \phi)$ and $int_{\mu M}(M, N) = (\phi, \phi)$.

Theorem 3.19. *In a binary supra M-topological space* $(M, N, b_{\mu M})$, *if* $(M, N) \subseteq (M, N)$ *then prove the following,*
(i) $cl_{\mu M}(M, N)$ *is the smallest binary supra closed multiset containing (M,N).*
(ii) (M,N) is a binary supra closed multiset in $(M, N, b_{\mu M})$ *if and only if* $(M, N) = cl_{\mu M}(M, N)$.
Proof. (i) Let $\{(M_\alpha, N_\alpha) : \alpha \in \Delta\}$ be the collection of all binary supra closed multiset containing (M,N). Then $(O, P) = \bigcap\{(M_\alpha, N_\alpha) : \alpha \in \Delta\}$ is a binary supra closed multiset. Since each (M_α, N_α) is a superset of (M,N) that is (M,N) contained in their intersection. Therefore $(M, N) \subseteq (O, P)$ that is $(O, P) \subseteq (M_\alpha, N_\alpha)$ for each $(x, y) \in \Delta$, and hence (O, P) is the smallest binary supra closed multiset containing (M,N). Therefore, $cl_{\mu M}(M, N)$ is the smallest binary supra closed multiset containing (M,N).

(ii) If (M,N) is a binary supra closed multiset in $(M, N, b_{\mu M})$, then $(M, N) \subseteq cl_{\mu M}(M, N)$. Conversely, if $cl_{\mu M}(M, N) = (M, N)$, then (M,N) is a binary supra closed multiset because $cl_{\mu M}(M,N)$ is by definition 3.16 the smallest binary supra closed multiset containing (M,N) and $cl_{\mu M}(M,N) = (M,N)$.

Proposition 3.20. *Let (M_1, N_1) and $(M_2, N_2) \subseteq P^*(M) \times P^*(N)$ and $(M, N, b_{\mu M})$ is a binary supra multiset topological space. Then*

(i) $cl_{\mu M}(\phi, \phi) = (\phi, \phi)$ *and* $cl_{\mu M}(M, N) = (M, N)$.

(ii) $(M_1, N_1) \subseteq cl_{\mu M}(M_1, N_1)$.

(iii) $cl_{\mu M}(cl_{\mu M}(M_1, N_1)) = cl_{\mu M}(M_1, N_1)$.

(iv) $cl_{\mu M}(M_1, N_1) \cup (cl_{\mu M}(M_2, N_2) \subseteq cl_{\mu M}((M_1, N_1) \cup (M_2, N_2))$.

(v) $cl_{\mu M}(M_1, N_1) \cap (cl_{\mu M}(M_2, N_2) \subseteq cl_{\mu M}((M_1, N_1) \cap (M_2, N_2))$.

Proof.(i) By the theorem, (M_1, N_1) is a binary supra closed multiset in $(M, N, b_{\mu M})$ if and only if $(M_1, N_1) = cl_{\mu M}(M_1, N_1)$. Since both (ϕ, ϕ) and (M,N) are binary supra closed multiset, then $cl_{\mu M}(\phi, \phi) = (\phi, \phi)$ and $cl_{\mu M}(M, N) = (M, N)$.

(ii) According to the theorem, $cl_{\mu M}(M_1, N_1)$ is the smallest binary supra closed multiset containing (M_1, N_1), implying $(M_1, N_1) \subseteq cl_{\mu M}(M_1, N_1)$.

(iii) Let (M_1, N_1) be a binary supra closed multiset in $(M, N, b_{\mu M})$. By definition 3.16, $(M_1, N_1) = cl_{\mu M}(M_1, N_1)$, and $cl_{\mu M}(M_1, N_1)$ is also a binary supra closed multiset. Therefore, $cl_{\mu M}(cl_{\mu M}(M_1, N_1)) = cl_{\mu M}(M_1, N_1)$.

(iv) Consider $(M_1, N_1) \subseteq (M_1, N_1) \cup (M_2, N_2)$ and $(M_2, N_2) \subseteq (M_1, N_1) \cup (M_2, N_2)$. Thus, $cl_{\mu M}(M_1, N_1) \subseteq cl_{\mu M}((M_1, N_1) \cup (M_2, N_2))$ and $cl_{\mu M}(M_2, N_2) \subseteq cl_{\mu M}((M_1, N_1) \cup (M_2, N_2))$. Therefore, $cl_{\mu M}(M_1, N_1) \cup cl_{\mu M}(M_2, N_2) \subseteq cl_{\mu M}((M_1, N_1) \cup (M_2, N_2))$.

(v) For the intersection, let $(M_1, N_1) \cap (M_2, N_2) \subseteq (M_1, N_1)$ and $(M_2, N_2) \cap (M_2, N_2) \subseteq (M_1, N_1)$. Thus, $cl_{\mu M}((M_1, N_1) \cap (M_2, N_2)) \subseteq cl_{\mu M}(M_1, N_1)$ and $cl_{\mu M}((M_1, N_1) \cap (M_2, N_2)) \subseteq cl_{\mu M}(M_2, N_2)$. Hence, $cl_{\mu M}(M_1, N_1) \cap cl_{\mu M}(M_2, N_2) \subseteq cl_{\mu M}((M_1, N_1) \cap (M_2, N_2))$.

Theorem 3.21. *Let $(M, N, b_{\mu M})$ be a binary supra M-topological space, and let (M,N) be a subspace of $(M, N, b_{\mu M})$. Then*

(i) $int_{\mu M}(M, N)$ *is a binary supra open multiset.*

(ii) $int_{\mu M}(M, N)$ *is the largest binary supra open multiset contained in (M,N).*

(iii) (M,N) is a binary supra open multiset if and only if $int_{\mu M}(M, N) = (M, N)$.

Proof. (i) Every binary supra open multiset is a binary supra M-neighborhood of each of its points. Let $(m,n) \in int_{\mu M}(M,N)$ is a binary supra interior multiset of (M,N). Thus, there exists a binary supra open multiset (M_2,N_2) such that $(m,n) \in (M_2,N_2) \subseteq (M,N)$. Since (M_2,N_2) is binary supra open multiset, it is a binary supra M-neighborhood of each of its points, and hence (M,N), being a superset of (M_2,N_2), is also a binary supra M-neighborhood of each point of (M_2,N_2). Thus, every points of (M_2,N_2) is a binary supra interior multiset point of (M,N), implying $(M_2,N_2) \subseteq int_{\mu M}(M,N)$. Therefore, $(m,n) \in (M_2,N_2) \subseteq int_{\mu M}(M,N)$. It follows that every point in (M_2,N_2) is contained in $int_{\mu M}(M,N)$. Hence, $int_{\mu M}(M,N)$ is a binary supra M-neighborhood of each of its points and consequently a binary supra open multiset.

(ii) Let (M_2,N_2) be binary supra open multiset contained in (M,N), and let $(m,n) \in (M_2,N_2)$. Then $(m,n) \in (M_2,N_2) \subseteq (M,N)$. Since (M_2,N_2) is binary supra open, (M,N) is a binary supra M-neighborhood of $(m,n) \in (M_2,N_2)$, and hence (M,N) is a $int_{\mu M}(M,N)$. Since $(m,n) \in (M_2,N_2)$, it implies $(m,n) \in int_{\mu M}(M,N)$. Therefore, $(M_2,N_2) \subseteq int_{\mu M}(M,N)$, and from (i), is an binary supra open multiset. Thus, $int_{\mu M}(M,N)$ contains every binary supra open multiset (M_2,N_2) of (M,N), making it the largest such subset.

(iii) If $(M,N) = int_{\mu M}(M,N)$, then $int_{\mu M}(M,N)$ is a binary supra open multiset, implying (M,N) is also a binary supra open multiset. Conversely, if (M,N) is a binary supra open multiset, then by (i), $int_{\mu M}(M,N)$ is the largest binary supra open multiset of (M,N). Hence, $int_{\mu M}(M,N) = (M,N)$.

Theorem 3.22. *Let $(M,N,b_{\mu M})$ be a binary supra M-topological space and let (M_1,N_1) and (M_2,N_2) be any subset of (M,N), following by,*

(i) $int_{\mu M}(\phi,\phi) = (\phi,\phi)$.

(ii) $int_{\mu M}(M,N) = (M,N)$.

(iii) $int_{\mu M}(M_1,N_1) \cap (int_{\mu M}(M_2,N_2) \subseteq int_{\mu M}((M_1,N_1) \cap (M_2,N_2))$.

(iv) $int_{\mu M}(int_{\mu M}(M_1,N_1)) = int_{\mu M}(M_1,N_1)$.

(v) $int_{\mu M}(M_1,N_1) \cup (int_{\mu M}(M_2,N_2) \subseteq int_{\mu M}((M_1,N_1) \cup (M_2,N_2))$.

Proof. Let (M_1,N_1) be binary supra open multiset iff $int_{\mu M}(M_1,N_1) = (M_1,N_1)$.
(i) and (ii) Since both $(\emptyset,\emptyset)$ and (M,N) are binary supra open multiset, it follows that $int_{\mu M}(\emptyset,\emptyset) = (\emptyset,\emptyset)$ and $int_{\mu M}(M,N) = (M,N)$.

(iii) Consider $(M_1,N_1) \cap (M_2,N_2) \subseteq (M,N)$. Then $int_{\mu M}((M_1,N_1) \cap (M_2,N_2)) \subseteq$

$int_{\mu M}(M_1, N_1)$ and $int_{\mu M}((M_1, N_1) \cap (M_2, N_2)) \subseteq int_{\mu M}(M_2, N_2)$. Hence, $int_{\mu M}((M_1, N_1) \cap (M_2, N_2)) \subseteq int_{\mu M}(M_1, N_1) \cap int_{\mu M}(M_2, N_2)$.

(iv) Since, $int_{\mu M}(M_1, N_1)$ is a binary supra open multiset, hence $int_{\mu M}(int_{\mu M}(M_1, N_1)) = int_{\mu M}(M_1, N_1)$.

(v) Thus, $(M_1, N_1) \subseteq (M_1, N_1) \cup (M_2, N_2)$ and $(M_2, N_2) \subseteq (M_1, N_1) \cup (M_2, N_2)$. Therefore, $int_{\mu M}(M_1, N_1) \subseteq int_{\mu M}((M_1, N_1) \cup (M_2, N_2))$ and $int_{\mu M}(M_2, N_2) \subseteq int_{\mu M}((M_1, N_1) \cup (M_2, N_2))$. Hence, $int_{\mu M}(M_1, N_1) \cup int_{\mu M}(M_2, N_2) \subseteq int_{\mu M}((M_1, N_1) \cup (M_2, N_2))$.

4. Relations Between Binary Supra M-space

Definition 4.1. *A subset of (A,B) of $(M, N, b_{\mu M})$ is called*

(i) A binary supra pre-open multiset if $(A, B) \subseteq int_{\mu M}(cl_{\mu M}(A, B))$.

(ii) A binary supra pre-closed multiset if $cl_{\mu M}(int_{\mu M}(A, B)) \subseteq (A, B)$.

(iii) An binary supra α-open multiset if $(A, B) \subseteq cl_{\mu M}(int_{\mu M}(A, B))$.

(iv) An binary supra α-closed multiset if $cl_{\mu M}(int_{\mu M}(cl_{\mu M}(A, B))) \subseteq (A, B)$.

(vi) A binary supra rα -open multiset if $(A, B) = cl_{\mu M}(int_{\mu M}(A, B))$.

(v) A binary supra semi-open multiset if $(A, B) \subseteq cl_{\mu M}(int_{\mu M}(A, B))$.

(vi) A binary supra semi-closed multiset if $int_{\mu M}(cl_{\mu M}(A, B)) \subseteq (A, B)$.

Remark 4.2. *Binary supra α-open multiset forms a binary supra m-topology. Every binary supra open multiset is a binary supra α-open multiset but the converse is not true.*

Theorem 4.3. *Let subset (A,B) be a $(M, N, b_{\mu M})$ is binary supra semi-open multiset if and only if $(A, B) = cl_{\mu M}(int_{\mu M}(A, B))$.*
Proof. Assume (A,B) that subset in every open bsm is know as binary supra semi-open multiset, and the complement of a bsm semi-open is binary supra semi-closed multiset. A binary supra semi-open multiset satisfies $(A, B) \subseteq int_{\mu M}(A, B)$. A binary supra closed multiset is defined by $(A, B) \subseteq cl_{\mu M}(A, B) \subseteq b^c_{\mu M}$. Since $(A, B) \subseteq cl_{\mu M}(int_{\mu M}(A, B))$, it follows that $(A, B) = int_{\mu M}(A, B)$.

Theorem 4.4. *A subset (A,B) of the binary supra semi-topological space $(M, N, b_{\mu M})$ is classified as binary supra semi-closed if and only if $int_{\mu M}(cl_{\mu M}(A, B)) = (A, B)$.*
Proof. Let (A,B) be a subset of (M,N). The complement of an supra binary open multiset is binary supra closed multiset, which means that a binary supra

semi-closed multiset satisfies $(A,B) \subseteq cl_{\mu M}(A,B) \subseteq b^c_{\mu M}$. Conversely, the complement of a binary supra closed mset is binary supra semi-open multiset, given by $(A,B) \subseteq int_{\mu M}(A,B) \subseteq b^c_{\mu M}$. Thus, we have $int_{\mu M}(cl_{\mu M}(A,B)) \subseteq (A,B) \subseteq b^c_{\mu M}$.

Example 4.5. Let $M = \{1/x, 2/y\}, N = \{3/c, 6/d\}$ be a binary supra M-topology, i.e., $b_{\mu M} = \{(\phi,\phi), (\{M,N\}), (\{\phi, 1/y\}), (\{1/x, 2/y\}), (\{2/c\}, \{3/d\}), (\{5/d\}, M_2)\}$ then $b_{\mu M}(\alpha) = \{(\phi,\phi), (\{M,N\}), (\{\phi, 1/y\}), (\{1/x\}), (M_1, \{3/d\}), (\{5/d\}, M_2)\}$. Here $b_{\mu M}(\alpha)$-open multiset need not to be binary supra open multiset.

Theorem 4.6. *Every binary supra closed multiset in (M,N) is binary supra α-closed multiset.*

Proof. Let (A,B) be a binary supra closed multiset in (M,N). Let (G,H) be any binary supra α-open multiset in (M,N) Such that $(A,B) \subseteq (G,H)$. Since (A,B) is binary supra closed multiset , we have $cl_{\mu M}(A,B) = (A,B)$, and $\alpha cl_{\mu M}(A,B) \subseteq cl_{\mu M}(A,B) \subseteq (G,H)$. Hence (A,B) is binary supra α-closed multiset in (M,N).

Theorem 4.7. *Every binary supra α -closed multiset in (M,N) is binary supra rα-closed multiset.*

Proof. Let (A,B) be a binary supra α-closed multiset in binary supra multiset topological space (M,N). Let (U,V) be an binary supra rα-open multiset in (M_1, M_2) such that $(A,B) \subseteq (U,V)$. Since (A,B) is binary supra α-closed multiset, and $\alpha cl_{\mu M}(A,B) = (A,B) \subseteq (U,V)$. Therefore $\alpha cl_{\mu M}(A,B) \subseteq (U,V)$. Hence (A,B) is binary supra rα-closed multiset.

Theorem 4.8. *Every binary supra open multiset (M,N) is binary supra α-open multiset.*

Proof. Let (A,B) be a binary supra closed multiset in (M,N). Let (S,T) be any binary supra α-closed multiset in binary supra multiset topological space. Such that $(A,B) \subseteq (S,T)$. Since (A,B) is binary supra open multiset, and $cl_{\mu M}(A,B) = (A,B)$, Since $\alpha int_{\mu M}(A,B) \subseteq int_{\mu M}(A,B) \subseteq (S,T)$. Hence (A,B)is binary supra α-open multiset (M,N).

Theorem 4.9. *Every binary supra α-open multiset in (M,N) binary supra rα-open multiset.*

Proof. Let (A,B) be a binary supra α-closed multiset in (M,N). Let (U,V) be an binary supra rα-open multiset in binary supra topological space such that $(A,B) \subseteq (U,V)$. Since (A,B) is binary supra α -open multiset and $\alpha int_{\mu M}(A,B) = (A,B) \subseteq (U,V)$. Therefore $\alpha int_{\mu M}(A,B) \subseteq (U,V)$. Hence (A,B) is binary supra rα-open multiset.

Theorem 4.10. *Every binary supra open multiset (M,N) is binary supra pre-open multiset (M,N).*

Proof. Since (A,B) is binary supra open multiset in $(M_1, M_2, b_{\mu M})$, such that (G,H) be any binary supra pre-closed multiset in (M,N). Such that $(A, B) \subseteq (G, H)$. Thus $(int_{\mu M}(cl_{\mu M})(A, B)) \subseteq int_{\mu M}(A, B)$, therefore binary supra pre-open multiset of $int_{\mu M}(A, B) \subseteq$ (G,H).
Hence $(int_{\mu M}(cl_{\mu M}(A, B)) \subseteq$ (G,H).

5. Conclusion

This paper introduced the concept of binary supra multiset topology, established its characterizations, and analysed its key properties. The relationship with existing multiset topological structures was also examined, laying a basis for further research.

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